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AAMT Connect Project Modelling and Applications

# **AAMT Connect Project**

## **Modelling and Applications**

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**Webinar Launch**

**July 2014**

Slide #2

## Modelling, Applications, and Numeracy in the Australian Context

From the Australian Curriculum Assessment and Reporting Authority, 2010

- *Mathematics* aims to ensure that students are confident, creative users and communicators of mathematics, able to *investigate, represent and interpret situations in their personal and work lives and as active citizens.*
- From the USA: (Common Core State Standards Initiative, 2012):
- Mathematically proficient students can apply the mathematics they know to *solve problems arising in everyday life, society, and the workplace.*
- Similar statements are put out from time to time by the OECD
- Subsequent curriculum documents provide little direct advice for achieving these purposes
- The Connect project provides one avenue for addressing this need

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## Numeracy

# Numeracy

Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

(OECD 2010, p. 4)

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Australian Curriculum

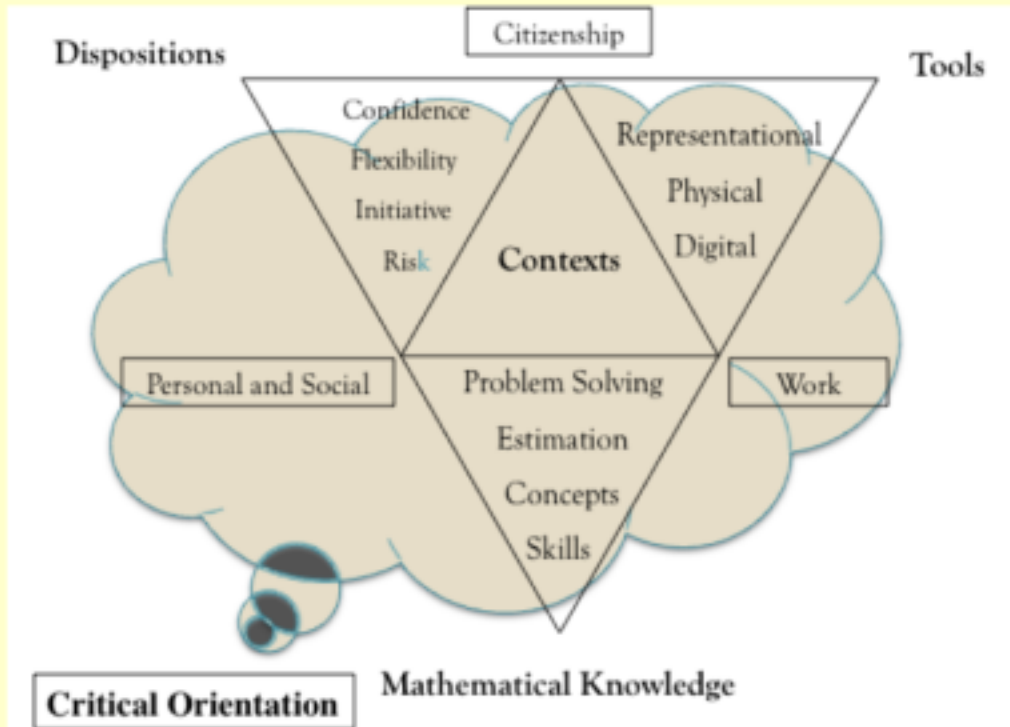
## Australian Curriculum

Students become numerate as they develop the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.

(Australian Curriculum, Assessment and Reporting Authority 2014, p. 13)

A Model of Numeracy for the 21st Century

# A Model of Numeracy for the 21<sup>st</sup> Century



A model for numeracy in the 21<sup>st</sup> century (Goos, Dole & Geiger, 2011)

21st Century Numeracy

## 21<sup>st</sup> Century Numeracy

mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.

Critical Aspects of Numeracy

## Critical Aspects of Numeracy

The empowered learner will not only be able to pose and solve mathematical questions (mathematical empowerment), but also will be able to understand and begin to answer important questions relating to a broad range of social uses and abuses of mathematics (social empowerment). Many of the issues involved will not seem primarily to be about mathematics, just as keeping up to date about current affairs from reading broadsheet newspapers is not primarily about literacy. Once mathematics becomes a '*thinking tool*' for viewing the world critically, it will be contributing to both the political and social empowerment of the learner, and hopefully to the promotion of social justice and a better life for all.

(Ernest 2002, p. 6)



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## STARTING POINTS ORIENTATION

### Mathematical Modelling as real world problem solving

- **Starting points** begin from the assumption that mathematics is everywhere in the world around us - the challenge is to identify its presence, access it, and apply it productively.

The approach aims to do two things

- Firstly, to provide a (tested) systematic approach to addressing problems located in real world settings
- Secondly, by this means help students develop skills of real world problem solving.
- The latter means that they can address productively problems set by others, but also (in keeping with the stated curriculum intentions), learn to identify and address problems that matter to them. (Serves all three curriculum purposes.)
- The solution of a problem with roots in the world outside the classroom will often provide ideas and examples that can be tailored to a variety of additional pedagogical ends (e.g. enhance mathematical curricular content.)
- Traditional word problems are not considered to advance these purposes

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**Real World Problems – a method of attack**

1. Identify a (real world) problem setting
2. Specify a related mathematical question
3. Formulate a mathematical model to address the question (involves making assumptions, choosing variables, estimating magnitudes of inputs etc)
4. Solve the mathematics
5. Interpret the mathematical results in terms of their real world meanings
6. Evaluation - make a judgment as to the adequacy of the solution to the original question
7. Report a success, make adjustments and try for a better solution, refine the problem, identify and pursue further questions suggested by the problem setting/solution

- The process is rarely linear as checking, correcting, and reflecting means that there is usually movement to and fro between the stages
- Evaluation often indicates a need to refine, rework or amend original problem (cyclic)
- A problem can be addressed at different levels - for example with arithmetic or algebra.
- Evaluation typically involves both mathematical and contextual features. Is the mathematics correct? ? Does the solution provide insights in the real setting?
- Sometimes a solution cannot be tested against real world data – futures modelling
- A successful outcome can sometimes be measured by the degree of enhanced insight it provides into a problem area, and how it suggests further work.
- Step 6 (Evaluation) does not always play as central a role in numeracy problems

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**Problem context (environmental) – Saving Trees**



Today, 90% of paper pulp is created from wood. Paper production accounts for about 35% of felled trees, and represents 1.2% of the world's total economic output (Wikipedia).

We can identify several mathematical problems of interest. For example:

**Mathematical Problem: How much timber is needed to provide a years circulation of a newspaper?** (*much, year, and circulation are quantitative terms*)

Obviously the problem can be tailored precisely to any community in which it is to be used (local paper etc).

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**Estimate the number of trees needed to print a year's supply of a newspaper of the size and circulation of the Brisbane daily - Courier Mail/ Sunday Mail.**

**Disclaimer:** This does not necessarily represent the *specifics* of the paper used as a basis for estimation – specifics of production are not known.

**Assumptions**

- No recycled paper is used
- Newspaper has the same style and size week by week
- Average weekly circulation figures apply across a year
- Ignore inserts/magazines (provides for a second modelling cycle)
- Other...

**Identify required information\***

- page dimensions
- area density of newsprint
- pages per year
- trees per tonne of newsprint

\* Need not all happen at the start – can emerge progressively throughout the solution process

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**Initial Data:** By **measuring** and **averaging** (e.g. groups of 2 or 4)

**Page (and margin) dimensions** for the Brisbane tabloid (Courier Mail) and the national broadsheet (Australian\*).

	Width (cm)	Height (cm)	Top margin (cm)	Bottom margin (cm)	Left margin (cm)	Right margin (cm)
Courier Mail	29	40	1.8	1.6	1.8	1.8
Australian	41.7	58	1.3	1.3	1.3	2.3

**Number of pages:** (direct observation)

Sun (96); Mon (64); Tue (56); Wed (80); Thu (80); Fri (112); Sat (96)

**Daily circulation figures:** available on **websites** that are continually updated or replaced. (**Technology**)

In 2013 the *Courier Mail* circulation was listed as follows:

**Average circulation** Mon – Fri 185 770; Sat: 237 798; Sunday Mail: 438 994  
(not allowing for additional copies printed)\*

*\*Issues of approximation and rounding*

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**Estimating newsprint usage**

Courier Mail single copy (pages/week): 584

Area of page 1160 sq cm = 0.116 sq m

Circulation (pages/week) = 137 492 032

Total Area per Year = pages/week x weeks\*/year x area of page  
(No paper on Christmas day or Good Friday – does this matter?)

Area per year = 827 757 029 sq m (approximately)

**Conversion of area to weight (key move)**

If an electronic balance is available it is simplest to weigh some.

Alternative – search the Internet and locate a measure called: *Grammage*

Mass per unit area of all types of paper and paperboard is expressed in terms of grams per square metre (g/m<sup>2</sup>).

[http://www.ifra.com/website/Material.nsf/All/B8F2E0F0DC1CB591C12569F90055974F/\\$FILE/more\\_about\\_grammage.pdf](http://www.ifra.com/website/Material.nsf/All/B8F2E0F0DC1CB591C12569F90055974F/$FILE/more_about_grammage.pdf)



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**Assume a typical figure for grammage** (discussion) : say  $45 \text{ g/m}^2 = 0.045 \text{ kg/m}^2$

Courier/Sunday Mail equivalent for year weighs  $827\,757\,029 \times 0.045 = 37\,249\,066 \text{ kg} = 37\,249.1 \text{ tonne}$ .

**How many trees make a ton(ne) of paper? (Back to the Internet)**

An estimate (1992) based on a mixture of trees 40 feet tall and 6-8 inches in diameter, indicated it would take a rough average of 24 trees to produce a ton of newsprint.  
(Suggests the process is now about twice as efficient in using trees)

<http://conservatree.org/learn/EnviroIssues/TreeStats.shtml>

American source means the weight must be expressed in tons rather than tonnes. (**1 tonne = 0.98 ton**)

So for a paper with size and circulation similar to the Courier Mail/Sunday mail  
Estimate of number of trees used annually =  $0.98 \times 37\,249.1 \times 12 = 438\,050$   
trees (if no recycling takes place.)

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**Plantations: How much area is involved?**

**Internet source** gives the 'ideal' initial stocking rate for most eucalypts in South Australia as around 1000 stems per hectare. Typically seedlings are planted 2.5m apart along rows that are 4m apart.

[http://www.pir.sa.gov.au/\\_data/assets/pdf\\_file/0015/75021/FS16\\_Eucalypt\\_Seedling\\_Information\\_and\\_Planting\\_Tips.pdf](http://www.pir.sa.gov.au/_data/assets/pdf_file/0015/75021/FS16_Eucalypt_Seedling_Information_and_Planting_Tips.pdf)

Assuming a "square" distribution of trees:  $N = 438\,050$  trees and  $\sqrt{438\,050} = 662.0$

Area:  $(663 \times 2.5) \times (663 \times 4) = 4\,369\,210 \text{ (m}^2\text{)} = 436.9 \text{ hectares} \approx 4.4 \text{ sq km.}$

**Evaluating the solution: (precise outcomes are often commercial in confidence)**

- Carefully checking mathematical calculations for accuracy (with ballpark approximations) is a starting point. What about the real world context?
- Evaluating numerical accuracy is an issue with all futures modelling - population, climate change, oil reserves etc
- Can ask instead whether all important variables have been considered?
- What if we change assumptions – how sensitive are the outcomes?  
Where can we act most decisively?
- How has understanding of the issue been enhanced?

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- Problems like this one can be connected to curriculum statements about the importance of students able to use mathematics in their role as *responsible citizens*.
- Specifically, reflecting on the solution above can generate refinements and extensions that involve further modelling.
- Two are suggested below – one motivated by the observation that a lot of paper is used for margins that contain only blank space, the other by the practice of recycling for which information is available from internet sources

**Refinements and extensions**

**Margins**

Area of *printed part of page* =  $25.6 \times 36.4 = 931.8$  sq cm

Fraction 'wasted (blank) space' =  $(1160 - 931.8)/1160 = 19.7\%$

Annual weight not used for print =  $0.19 \times 36\,851 = 7249$  ton

*Trees used for margins (blank space)* =  $7249 \times 12 = 86,988$  ( $\approx 87,000$ )

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## Recycling

Australia leads the world in newsprint recycling

<http://www.proprint.com.au/News/266132,australia-leads-the-world-in-newsprint-recycling.aspx>

Australia's newsprint recycling rate is almost 10% higher than in Europe, according to a new report. The survey, commissioned by the Publishers National Environment Bureau (PNEB), showed that 78.7% of all Australian newsprint was recovered.

(The recycling rate is almost 10% higher than the average rate for newsprint in Europe, which is 68.9%.)

How many trees are saved annually by using 78.7% recycling of newsprint?

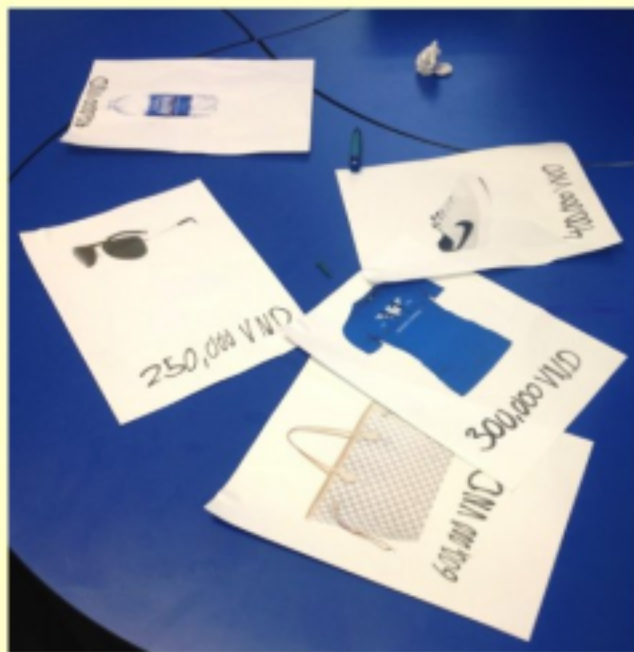
Trees saved by recycling =  $0.787 \times 438\,050 = 344\,745$  (approximately 3.5 sq km)

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### An Example of a Numeracy Problem

## An Example of a Numeracy Problem

Students were soon to depart on a school trip to Vietnam. Their teacher wished to assist her students to understand of how to manage their money. Bargaining in a market place and how to calculate conversions between Australian dollars and Vietnamese dong were activities students needed to accommodate.



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## A Second Numeracy Example

# A Second Numeracy Example

- The teacher's aim was to challenge her students to think about and understand the way the mass media make use of data and statistics to inform and/or manipulate opinion.
- The report also foreshadowed a change in the law that would include a clause that owners of dogs who attacked people would be barred from owning a dog for 10 years.

### LAW & ORDER

#### Pet bans, jail time for irresponsible dog owners under new government crackdown

PETER MCKELBUPOUGH • HERALD SUN • JULY 26, 2013 13:05:08



Figure 4: Headline of article related to potential new laws about dog attacks (Source: Herald Sun 2013)

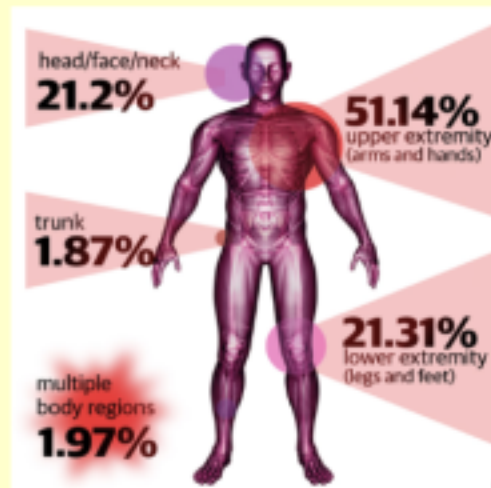


Figure 5: Graphical representation of the body parts most injured during dog attacks (Source: Herald Sun 2013)

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### Structured Problems

1. Find the area of newsprint in a page of a regular newspaper. What percentage of this is used for margins?
  2. What area of newsprint is used in a week? In a year?
  3. *Grammage* measures the number of grams per square metre ( $\text{g/m}^2$ ) of paper. See:  
[http://www.ifra.com/website/Material.nsf/All/B8F2E0F0DC1CB591C12569F90055974F/\\$FILE/more\\_about\\_grammage.pdf](http://www.ifra.com/website/Material.nsf/All/B8F2E0F0DC1CB591C12569F90055974F/$FILE/more_about_grammage.pdf)  
Assume a typical figure for grammage: say  $45 \text{ g/m}^2$  and calculate the weight of newsprint (in tonnes) needed in a year.
  4. The website below gives information about the number of trees needed for a (ton) of newsprint. The use of an American source means the weight measure must be converted ( $1 \text{ tonne} = 0.98 \text{ ton}$ ).  
<http://conservatree.org/learn/EnviroIssues/TreeStats.shtml>  
Using the suggested figure of 12 trees per ton estimate the number of trees needed for a years production of the newspaper.
  5. If trees are planted in a square pattern with 2.5 m between seedlings what area of plantation would this be? What area of plantation is needed for margins?
  6. Australia leads the world in newsprint recycling  
<http://www.proprint.com.au/News/266132.australia-leads-the-world-in-newsprint-recycling.aspx>  
How many trees would be saved by using the given figure of 78.7% for recycled paper?
- Structured problems allow for discussion and input but also supply some of the strategic decision making
  - Hence structured problems do not serve the second objective (helping students to become able to Independently apply mathematical knowledge) as well.
  - The structuring can be used as a basis for providing hints when a more open problem is posed

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Slide #1

**Sample ideas for Modelling Problems**

- **Fermi Problems**
  1. Estimate the number of tennis balls that would fill a classroom.
  2. Estimate how many dentists Toowoomba needs. Now? In approximately 20 years (2031)?
- The Trans-Alaska Pipeline is 1,300 kilometres long, 122 centimetres in diameter, and can carry 350 million litres of crude oil each day. Estimate how much oil is pumped through the Trans- Alaska Pipeline (in cubic metres/hr; in litres/min; in barrels/day). How fast is the oil being pumped along the pipeline in km/hr? (1 barrel of oil = 159 litres (approx)).
- On April 23 2013 Australia's population was increasing by one person every minute and 23 seconds. What does this mean for the future size of our population?
- How long would it take to evacuate Australia's largest residential building (Q1 on the Gold Coast) in case of fire?
- What is the probability that a top line tennis player wins a service game from 15 – 40?
- What is the maximum number of vehicles per hour to be expected to pass through a traffic tunnel lane?
- *TAILGATERS are causing more than 3000 crashes a year in Queensland - and those are just the ones that are reported.* (Courier Mail: November 02, 2009.) Investigate how separation distance between vehicles contributes to tailgating accidents.
- Morgan Spurlock's film *Supersize Me* mapped a 30-day period during which he ate only [McDonald's food](#) consuming an average of 5,000 [kcal](#) per day, while maintaining a sedentary lifestyle. His weight increased from 84 kg to 95.1 kg over the 30 day period. Develop a mathematical model that describes the weight gain experienced by Morgan.
- Mozart's compositions are assigned Kochel Numbers which relate to the date of their publication. Find a formula that can be used to assign a Kochel Number to a recently discovered composition by Mozart completed in April 1784.

Slide #1

## Maths in Action Forum Q & A Session (post webinar)

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Forum	Topics	Posts	Last reply
<b>Starting Points</b> A forum for general discussion that does not fit into other specific forums. Introduce yourself, ask a question, or make a comment. Moderator:	2	2	<b>Exercises Real World Problems</b> 07/07/2014 12:08 am Saver Huanou
<b>Young Learners</b> Maths in Action Forum discussion designed for teachers in primary school settings. Moderator:	0	0	

Maths in Action Forum – You will need to be a member to login  
<http://connectwith.mathsinaction.aamt.edu.au/Forums>

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Exploring Real World Problems

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Author	Message
Renee Hoareau (online)	Mon 07 July 2014 12:06:08 am Teachers are invited to respond to the forum topic conducted by Peter and Vileca and to discuss strategies that generate active learning.

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